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THE FINITE VELOCITY OF HEAT PROPAGATION FROM THE VIEWPOINT OF THE KINETIC THEORY

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Abstract—An approach is suggested for solving the problem on finite velocity of heat propagation from the viewpoint of the kinetic theory. Interrelation has been established between this problem and the finite time for the distribution function to approach equilibrium.

An approximation for the collision term in the kinetic Boltzmann equation has been found capable of providing the finite time for the equilibrium to be developed in an insulated system. By solving the Boltzmann equation, the distribution function is calculated to a first approximation through the agency of which the power–heat-conduction law is found, whose heat conduction equation describes the finite propagation velocity of thermal disturbances.

NOMENCLATURE

f,	distribution function of gas molecules;
f_0,	equilibrium distribution function;
$(\partial f/\partial t)_{st}$,	collision term in the Boltzmann kinetic equation;
$\mathbf{v}$,	molecule velocity;
m,	molecule mass;
$\mathbf{F}$,	force acting on a molecule;
t,	time;
$\mathbf{r}$,	point vector in coordinate space;
x, y, z,	Cartesian coordinates;
v_x, v_y, v_z,	molecule velocity projections onto Cartesian axes;
v, θ, φ,	spherical coordinates in velocity space;
n,	density of number of molecules;
k,	Boltzmann constant;
T,	temperature;
$\mathbf{q}$,	heat flux vector;
ρ,	density;
C_p,	specific heat;
p,	pressure;
$\Gamma(x)$,	gamma-function.

1. INTRODUCTION

AMONG the challenges of the transfer phenomena theory, there is a paradox of the infinite velocity of propagation of disturbances which, as is known, lies in the fact that the classical parabolic-type transfer equations describe propagation of disturbances in such a way that the effect of any disturbance being localized at the initial time instant in some space area, the next, whatever infinitesimal instant, extends over the whole unlimited space [1–4]. That is, propagation of disturbances, thermal ones in particular, occur with an infinite velocity. The first work aimed at settling this paradox appears to be the paper by Zel'dovich and Raizer [4] in which the set goal is accomplished by adopting a power-type dependence of heat capacity and thermal conductivity of the medium on temperature. Now, alongside this approach, wide use is made, primarily due to the efforts of the late A. V. Luikov, of a hyperbolic-type equation for description of heat conduction aimed at the same objective [1–3].

In this paper, another approach is employed based on the transfer processes being considered from the viewpoint of the kinetic theory. As a result, relationship between the above paradox and still another infinity observed in the kinetic theory of gases is rather accurately traced. By the second infinity the fact is meant that the kinetic Boltzmann equation yields time-exponential approach of the distribution function to its equilibrium value $[f-f_0 \sim \exp(-\lambda t)]$, i.e. speaking in general, an infinite time for equilibrium to set in. The main achievement of the present work is an established fact that a natural requirement for the distribution function to reach its equilibrium value in a finite time leads to transfer equations that describe propagation of disturbances with finite velocity.

Consider the Boltzmann equation for the distribution function $f(\mathbf{r}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\frac{\mathbf{F}}{m}\frac{\partial f}{\partial \mathbf{v}}=\left(\frac{\partial f}{\partial t}\right)_{st}. \tag{1.1}$$

The known approximations of the collision term in the kinetic Boltzmann equation (1.1), as for example,

$$\left(\frac{\partial f}{\partial t}\right)_{st}=-\frac{(f-f_0)}{\tau} \tag{1.2}$$

lead to a situation when the distribution function for a closed system tends with time from the initial value $\tilde{f}$ to its equilibrium value f_0 asymptotically by the exponential law

$$f-f_0=(\tilde{f}-f_0)\exp\left(-\frac{t}{\tau}\right) \tag{1.3}$$

i.e. generally speaking, equilibrium sets in after a time interval equal to infinity. It is known that use of the distribution function from the Boltzmann equation having a collision term (1.2) to describe the transfer processes leads to the transfer equations yielding an infinite velocity propagation of disturbances [5, 6].

2. FINITE TIME OF EQUILIBRIUM SETTING

Let us require for the distribution function to attain its equilibrium value in a finite time, which is physically more justifiable. To achieve this aim, expression (1.2) may be generalized by the following nonlinear approximation of the collision term

$$\left(\frac{\partial f}{\partial t}\right)_{st} = -\frac{f_0^{2k}(f-f_0)^{1-2k}}{\tau} \qquad \tfrac{1}{2} > k \geqslant 0 \quad (2.1)$$

i.e. we assume that at least at small deviations from equilibrium, the collision term is proportional to the deviation whose power is less than unity. Then the approach of the distribution function to the equilibrium value is governed by

$$\frac{\partial f}{\partial t} = -\frac{f_0^{2k}(f-f_0)^{1-2k}}{\tau} \quad (2.2)$$

and yields the law of this process as

$$f-f_0 = (\tilde{f}-f_0)\left[1-2k\frac{f_0^{2k}}{\tau(\tilde{f}-f_0)^{2k}}t\right]^{1/2k}. \quad (2.3)$$

Thus, it follows from (2.3) that in a finite time interval

$$t = \tau_0 \equiv \frac{(\tilde{f}-f_0)^{2k}\tau}{2kf_0^{2k}}$$

the distribution function takes exactly an equilibrium value being kept thereafter. In the domain of the real numbers, the function (2.3) is determined for the values of the argument $t \leqslant \tau_0$. At a point $t = \tau_0$, it turns to zero and by definition keeps this value at $t > \tau_0$. Note, that with restrictions imposed on k in (2.1), the first derivative function $f-f_0$ at the point $t = \tau_0$ turns to zero as well.

If τ_e is a time interval during which deviation of the function from an equilibrium value decreases e times relative to the initial one, then

$$\tau_0 = \frac{\tau_e}{1-\exp(-2k)}. \quad (2.4)$$

In the limit at $k \to 0$, equation (2.3) passes into (1.3), while $\tau_0 \to \infty$.

3. POWER LAW OF HEAT CONDUCTION

Now we shall show the effect which the form of the collision term (2.1) has in the studies of the transfer processes. Consider heat conduction in a gas whose temperature T is not a very strongly varying function of coordinates, so that for zero approximation we may take a local function of the Maxwellian distribution [5, 6], as is usually done

$$f_0(\mathbf{r}, \mathbf{v}, t) = n\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right). \quad (3.1)$$

Then, in the next approximation the distribution function will be determined by solving the kinetic Boltzmann equation (1.1) with the collision term (2.1) [5, 6]

$$\mathbf{v}\frac{\partial f_0}{\partial \mathbf{r}} = -\frac{f_0^{2k}(f-f_0)^{(1-2k)}}{\tau}. \quad (3.2)$$

To simplify subsequent calculations, we assumed a temperature distribution in a gas to be independent of time and the molecules to be unaffected by the external forces $\mathbf{F} = 0$.

From equation (3.2) we have

$$\begin{aligned} f &= f_0 - \left[\frac{\tau}{f_0^{2k}}\mathbf{v}\frac{\partial f_0}{\partial \mathbf{r}}\right]^{1+2k/(1-2k)} \\ &= f_0 - \left[\frac{\tau}{f_0^{2k}}\frac{\partial f_0}{\partial T}\mathbf{v}\frac{\partial T}{\partial \mathbf{r}}\right]^{1+2k/(1-2k)} \\ &= f_0 - \tau^{1/(1-2k)}\left[\frac{1}{T}v_i\frac{\partial T}{\partial x_i}\left(\frac{mv^2}{2kT}-\frac{5}{2}\right)\right]^{1/(1-2k)} \\ &\qquad \times n\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right). \end{aligned} \quad (3.3)$$

The distribution function being known, we can calculate the heat flux, which by definition [5, 6] is equal to

$$\mathbf{q} = \int d^3v\left(\frac{mv^2}{2}\right)\mathbf{v}f. \quad (3.4)$$

If temperature is distributed in a fluid in such a way that it is a function of only one coordinate, say x, then from equation (3.4): $q_y = q_z = 0$

$$q_x = -\lambda_H\left(\frac{\partial T}{\partial x}\frac{\partial T}{\partial x}\right)^l\frac{\partial T}{\partial x}, \quad l = \frac{k}{1-2k} \quad (3.5)$$

$$\lambda_H = -\frac{\tau^{1/(1-2k)}mn}{2T^{1/(1-2k)}}\left(\frac{m}{2\pi kT}\right)^{3/2}\int d^3v\, v^2 v_x^{2(1-k)/(1-2k)} \times\left[\frac{mv^2}{2kT}-\frac{5}{2}\right]^{1/(1-2k)}\exp\left(-\frac{mv^2}{2kT}\right). \quad (3.6)$$

To obtain a generalized law, consider an additional number of particular cases:

(a) $T = T(x, y)$, $\quad k = 1/3$,

$$q_x = -462\frac{\tau^3nk^3}{m^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\frac{\partial T}{\partial x},$$

$$q_y = -462\frac{\tau^3nk^3}{m^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\frac{\partial T}{\partial y},$$

$$q_z = 0 \quad (3.7)$$

(b) $T = T(x)$, $\quad k = 1/4$,

$$q_x = -27\frac{\tau^2nT^{1/2}k^{5/2}}{m^{3/2}}\left|\frac{\partial T}{\partial x_z}\right|\frac{\partial T}{\partial x},$$

$$q_y = q_x = 0, \quad (3.8)$$

(c) $T = T(x, y)$, $\quad k = 2/5$,

$$q_i = -\lambda_H\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]^2\frac{\partial T}{\partial x_i}, \quad q_z = 0,$$

$$\lambda_H = \frac{25}{2^8}\frac{mn\tau^5}{T^5}\left(\frac{m}{2kT}\right)^{3/2}\int_0^\infty dv\left(\frac{mv^2}{2kT}-\frac{5}{2}\right)^5 \times v^6\exp\left(-\frac{mv^2}{2kT}\right). \quad (3.9)$$

Generalizing formulas (3.5)–(3.9), we write:

$$\mathbf{q} = -\lambda_H[(\nabla T)^2]^l\nabla T, \quad \lambda_H = \lambda_H(T, p\ldots) > 0. \quad (3.10)$$

Thus, the heat conduction law is the power law (3.10), and the heat-conduction equation will have the form

$$\rho c_p \frac{\partial T}{\partial t} = -\text{div}\,\{\lambda_H[(\nabla T)^2]^l \nabla T\}. \tag{3.11}$$

Or in index notation

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i}\left\{\lambda_H \left(\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}\right)^l \frac{\partial T}{\partial x_i}\right\} \tag{3.12}$$

summation is carried out from 1 to 3 over repeating indices.

In the presence of heat sources, equation (3.11) is written as:

$$\rho c_p \frac{\partial T}{\partial t} = \text{div}\,\{\lambda_H[(\nabla T)^2]^l \nabla T\} + Q$$

where Q is the quantity of heat absorbed or released by these sources per unit time in a body unit volume.

In a one-dimensional case, the temperature being a function of only one space coordinate x, equation (3.12) takes on the form

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left\{\lambda_H \left[\left(\frac{\partial T}{\partial x}\right)^2\right]^l \frac{\partial T}{\partial x}\right\}. \tag{3.13}$$

If a new thermal conductivity coefficient, λ_H, can be regarded as a constant value, equation (3.12) gets a most simple form:

$$\frac{\partial T}{\partial t} = \kappa_H \frac{\partial}{\partial x}\left\{\left[\left(\frac{\partial T}{\partial x}\right)^2\right]^l \frac{\partial T}{\partial x}\right\}. \tag{3.14}$$

Here, a new term for thermal diffusivity,

$$\kappa_H = \frac{\lambda_H}{\rho c_p},$$

is introduced.

It is the above equation (3.14) to whose solution, provided κ_H and n are constant values, that the remainder portion of this paper will be devoted. By solving specific problems, illustration will be provided for the existence of the finite velocity of propagation of thermal disturbances (see also [7]).

4. PROPAGATION OF HEAT FROM A PLANE INSTANTANEOUS SOURCE

At the initial time instant, $t = 0$, in the plane $x = 0$ a quantity of heat released related to unit area, Q. The remaining space has a constant temperature, $T = T_0$. Find spatial distribution of temperature at subsequent time instants.

The problem stated is described by equation (3.14) subject to the initial conditions

$$t = 0: \quad \begin{cases} T = \infty & \text{at} \quad x = 0 \\ T = T_0 & \text{at} \quad x \neq 0 \end{cases} \tag{4.1}$$

the total quantity of heat in the space remaining constant

$$\int_{-\infty}^{\infty} \rho c_p (T - T_0)\,\text{d}x = Q = \text{const}$$

or

$$\int_{-\infty}^{\infty} (T - T_0)\,\text{d}x = A = \text{const} \tag{4.2}$$

Solution of equation (3.14) under initial conditions (4.1) is of the following form

$$T - T_0 = \frac{A[l/(l+1)]^{(2l+1)/2l}}{2^{1/2l}(2l+1)^{1/2l} a t^m} \times \left[\left\{(\xi_0^2)^{(l+1)/(2l+1)} - (\xi^2)^{(l+1)/(2l+1)}\right\}^{1/2}\right]^{(2l+1)/l} \tag{4.3}$$

where

$$\xi = x/at^m, \quad a = \kappa_H^{1/2(2l+1)} A^{l/(2+1)}, \quad m = 1/2(2l+1),$$

ξ_0 is an arbitrary constant.

Equation (4.3) gives spatial distribution of temperature bounded at any time instant by points $\xi = \pm\xi_0$. Outside these points the temperature is equal to T_0. Thus, the boundary of the region occupied at the given time instant by a thermal disturbance is determined from equality $\xi_{rp} = \pm\xi_0$ and with time this region extends according to the law

$$x_{rp} = \pm\xi_0 a t^m = \pm\xi_0 a t^{1/2(2l+1)}. \tag{4.4}$$

It is the velocity with which the boundary of the region, occupied by a thermal disturbance, moves which is the velocity of propagation of a thermal disturbance. It is defined by the derivative of $\partial x_{rp}/\partial t$ and is equal to

$$v_T = \frac{\partial x_{rp}}{\partial t} = \pm\frac{\xi_0 a}{2(2l+1)} t^{-(1+4l)/(2+2l)}. \tag{4.5}$$

At $t \to \infty$, we get $v_T \to 0$.

Note that at points $\xi = \pm\xi_0$ the first derivative of temperature with respect to the coordinate, $\partial T/\partial x$, turns to zero.

Qualitative representation of the temperature distribution at some time instant is shown in Fig. 1. The dashed line gives temperature distribution determined by the linear heat-conduction equation.

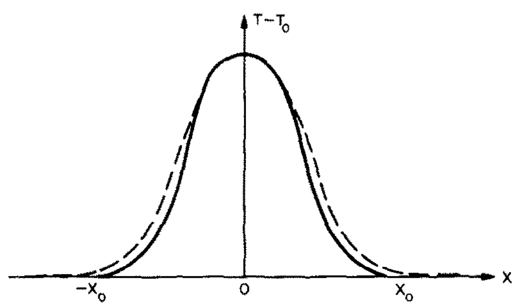

FIG. 1. Qualitative representation of the spatial temperature distribution from an instantaneous point source at time instant t on the basis of the nonlinear heat-conduction law (solid line) and the linear Fourier law (dashed line). Points $x_0 = \pm at^m\xi_0$ define the boundary of the region occupied by a thermal disturbance at the given time instant.

The value of the arbitrary constant ξ_0 is defined from the condition that the total quantity of heat in space is constant

$$\int_{-\infty}^{\infty} (T - T_0)\,\text{d}x = \int_{-\xi_0}^{\xi_0} (T - T_0)\,\text{d}x = A$$

and turns to be equal to

$$\xi_0 = (l+1)^{(1+4l)/(2+4l)} 2^{1/2(2l+1)} (2l+1)^{(1-2l)/2(2l+1)} \times l^{-1/2} \left[\frac{\Gamma\left(\frac{1}{2l} + \frac{2l+1}{2l+2} + 2\right)}{\Gamma\left(\frac{1}{2l}+2\right)\Gamma\left(\frac{2l+1}{2l+2}\right)} \right]^{l/(2l+1)}. \quad (4.6)$$

At small values of the exponent $l(l \to 0)$, we obtain

$$\kappa_H \to \kappa, \quad a \to (\kappa_H)^{1/2} \to \kappa^{1/2},$$

$$m \to 1/2, \quad \xi \to \frac{x}{(\kappa t)^{1/2}} \quad (4.7)$$

and formula (4.3) which gives spatial temperature distribution acquires the form:

$$T - T_0 = \frac{A}{(4\pi\kappa t)^{1/2}} \left[\left\{ 1 - 2l \left(\frac{x}{(4\kappa t)^{1/2}} \right)^2 \right\}^{1/2} \right]^{1/l}. \quad (4.8)$$

At $l \to 0$

$$\lim_{l \to 0} (T - T_0) = \frac{A}{(4\kappa\pi t)^{1/2}} \exp\left(-\frac{x^2}{4\kappa t} \right),$$

which is consistent with the linear heat-conduction theory [1, 2].

With $l \ll 1$ the boundary of the region occupied by a thermal disturbance is determined by:

$$x_{rp} = \pm \left(\frac{2\kappa}{l} \right)^{1/2} t^{1/2}. \quad (4.9)$$

And the propagation velocity of a thermal disturbance, by

$$v_T = \frac{\partial x_{rp}}{\partial t} = \pm \left(\frac{\kappa}{2l} \right)^{1/2} t^{-1/2}. \quad (4.10)$$

This expression makes it evident that the velocity of propagation of a thermal disturbance vanishes with time in proportion to $t^{-1/2}$. Its infinite value at the initial time instant is due to the statement of the problem.

Expression (4.10) allows estimation of the possible values of the exponent l which determines deviation from linearity in the heat conduction law. Thermal diffusivity κ is of the order of $(10^{-4}$–$10^{-7})\text{m}^2/\text{s}$ depending on the type of the medium. According to the available data, the velocity of propagation of a thermal disturbance is of the order of 10^2m/s [1]. Once this velocity is assumed to correspond to the time instant $t = 10\,\text{s}$ at the onset of a disturbance, then l should be of the order

$$l \sim 10^{-9} - 10^{-12}. \quad (4.11)$$

This value is very small as compared to unity, i.e. deviation from the linearity in the heat conduction law is so small that for its experimental determination some special methods are required. This small value of l allows approximate formulae (4.7)–(4.10) to be automatically employed for calculations.

Note that solution (4.3) may be interpreted otherwise. Let the temperature distribution be prescribed by formulae (4.3) at the initial time instant $t = t_0$ between the points $x_0 = \pm \xi_0 a t_0^{1/2(2l+1)}$. At points $-x_0 \geqslant x \geqslant x_0$ the temperature is constant and equal to T_0. Then, expression (4.3) gives temperature distribution at any subsequent time instant $t > t_0$. The initial temperature distribution is spread over a space with a finite velocity of boundary motion of the region occupied by a thermal disturbance. This solution is free of infinities at the initial time instant.

It is also valid for the case when one half-space (for example, $x < 0$) is thermally insulated, and a thermal disturbance in the plane $x = 0$ propagates into the other half-space (for example, $x > 0$), since

$$(\partial T/\partial x)_{x=0} = 0.$$

5. HEAT PROPAGATION FROM A PLANE WITH INSTANTANEOUSLY CHANGED TEMPERATURE

Let at the initial time instant, $t = 0$, the plane temperature, $x = 0$, assume the value T_1, afterwards being kept indefinitely long. The RHS half-space $(x > 0)$ has the temperature T_2. It is in this half-space that heat starts propagating. Find a temperature distribution in it at subsequent time instants.

This process is also governed by equation (3.14) under the following initial and boundary conditions:

$$\text{At } t = 0: \quad \begin{cases} T = T_1 & \text{at} \quad x = 0 \\ T = T_2 & \text{at} \quad x > 0 \end{cases} \quad (5.1)$$

$$\text{At } t > 0: \quad T = T_1 \quad \text{at} \quad x = 0. \quad (5.2)$$

The solution of equation (3.14) under the initial and boundary conditions (5.1) and (5.2) is as follows:

$$\frac{T(\xi) - T_1}{T_2 - T_1} = \left[\frac{l}{2(2l+1)(l+1)} \right]^{1/2l} \times \int_0^{\xi} [(\xi_0^2 - \eta^2)^{1/2}]^{1/l} \mathrm{d}\eta \quad (5.3)$$

where

$$\xi = \frac{x}{at^m}, \quad m = \frac{1}{2(l+1)}, \quad a = \kappa_H^{1/2(l+1)} \theta^{l(l+1)}$$

$$\theta = |T_1 - T_2|$$

$$\xi_0^{(l+1)/l} = \frac{2\Gamma\left(\frac{1}{2l} + \frac{3}{2}\right)}{\Gamma\left(\frac{1}{2l}+1\right)\Gamma\left(\frac{1}{2}\right)} \left[\frac{2(2l+1)(l+1)}{l} \right]^{1/2l}. \quad (5.4)$$

Formula (5.3) yields solution to the stated problem within $0 \leqslant \xi \leqslant \xi_0$. At $\xi = \xi_0$, $T = T_2$. This point is the right boundary of the region occupied by a thermal disturbance

$$x_{rp} = \xi_0 a t^m = \xi_0 a t^{1/2(l+1)}. \quad (5.5)$$

The velocity of propagation of the thermal disturbance front is

$$v_T = \frac{\partial x_{rp}}{\partial t} = \frac{\xi_0 a}{2(l+1)} t^{-(2l+1)/(2l+2)}. \quad (5.6)$$

At $t \to \infty$, $v_T \to 0$. At a point $\xi = \xi_0$, the derivative of $(\partial T/\partial x) = 0$.

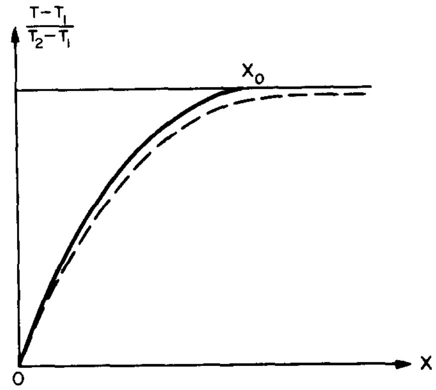

FIG. 2. Qualitative representation of the spatial temperature distribution at an instantaneous temperature change of the left half-space at time instant t on the basis of the nonlinear heat-conduction law (solid line) and the linear Fourier law (dashed line). The point $x_0 = at^m\xi_0$ determines the boundary of the region occupied by a thermal disturbance.

Qualitative representation of a temperature distribution at some time instant with $T_2 \neq T_1$ is given in Fig. 2. The dashed lines show temperature distribution obtained on the basis of the linear theory.

At small $l(l \to 0)$ we have:

$$\kappa_H \to \kappa, \quad a \to \kappa^{1/2}, \quad m \to 1/2, \quad \xi \to \frac{x}{(\kappa t)^{1/2}}$$

and solution (5.3) becomes

$$\frac{T(\xi)-T_1}{T_2-T_1} = \frac{1}{(\pi)^{1/2}} \int_0^{\xi} [(1-2l(\eta/2)^2)^{1/2}]^{1/l} \, d\eta \quad (5.7)$$

and

$$\lim_{l \to 0} \frac{T(\xi)-T_1}{T_2-T_1} = \frac{1}{(\pi)^{1/2}} \int_r^{\xi} \exp(-\eta^2/4) \, d\eta \equiv \operatorname{erf}\left(\frac{x}{(4\kappa t)^{1/2}}\right)$$

according to the linear theory [1, 2].

In this case too, at $l \ll 1$, the propagation velocity of thermal disturbances is equal to

$$v_T = \frac{\partial x_{rp}}{\partial t} = \left(\frac{\kappa}{2l}\right)^{1/2} \frac{1}{t^{1/2}} \quad (5.8)$$

just as in the first problem.

Note that the linear heat-conduction theory involves the concept of the isotherm velocity, which is also proportional to $t^{-1/2}$. It can be said that motion of the boundary of a region occupied by a thermal disturbance is motion of an isotherm with temperature of the medium surrounding this region.

Note in conclusion, that using the power form of the collision term in the Boltzmann equation, one may obtain a power rheological law as well as a power law of the combined heat and impulse transfer.

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PROPAGATION DE LA CHALEUR AVEC VITESSE FINIE DU POINT DE VUE DE LA THEORIE CINETIQUE

Résumé—On propose une approche permettant de résoudre le problème de la propagation de la chaleur avec vitesse finie du point de vue de la théorie cinétique. Un rapprochement a été effectué entre ce problème et celui d'un temps fini mis par la fonction de distribution pour atteindre l'équilibre.

Une approximation du terme de collision dans l'équation cinétique de Boltzmann s'est avérée capable de fournir le temps fini pour que s'établisse l'équilibre dans un système isolé. Par résolution de l'équation de Boltzmann, la fonction de distribution est calculée en première approximation et conduit à la loi puissance de la conduction thermique, l'équation de la chaleur correspondante décrit la vitesse de propagation finie des perturbations thermiques.

EINE BETRACHTUNG DER ENDLICHEN GESCHWINDIGKEIT DER WÄRMEAUSBREITUNG UNTER DEM GESICHTSPUNKT DER KINETISCHEN GASTHEORIE

Zusammenfassung—Es wird vorgeschlagen, das Problem der endlichen Geschwindigkeit der Wärmeausbreitung mit Hilfe der kinetischen Gastheorie zu lösen. Es wird ein Zusammenhang aufgezeigt zwischen diesem Problem und demjenigen der endlichen Zeit zum Erreichen des Gleichgewichtes für die Verteilungsfunktion.

Mit Hilfe einer Näherung für den Kollisionstherm in der Boltzmann-Gleichung konnte die endliche Zeit bis zur Einstellung des Gleichgewichtes in einem isolierten System angegeben werden. Die Lösung der Boltzmann-Gleichung ergibt eine erste Näherung für die Verteilungsfunktion, aus der ein Potenzgesetz für die Wärmeleitung aufgestellt werden kar n; diese V 'eitgleichung beschreibt die endliche Ausbreitungsgeschwindigkeit thermıscner Störungen.

КОНЕЧНАЯ СКОРОСТЬ РАСПРОСТРАНЕНИЯ ТЕПЛА С ТОЧКИ ЗРЕНИЯ КИНЕТИЧЕСКОЙ ТЕОРИИ

Аннотация — Предлагается подход к решению вопроса о конечной скорости распространения тепла с позиций кинетической теории. Установлена взаимосвязь между этим вопросом и конечным временем приближения функции распределения к равновесному значению.

Найдена аппроксимация столкновительного члена кинетического уравнения Больцмана, обеспечивающая конечное время установления равновесия в изолированной системе. Из решения уравнения Больцмана в первом приближении вычислена функция распределения и с ее помощью получен степенной закон теплопроводности, приводящий к уравнению теплопроводности, которое описывает процесс распространения тепловых возмущений с конечной скоростью.